The U-shaped distribution of globular cluster specific frequencies in a biased globular cluster formation scenario

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ABSTRACT

Using high-resolution numerical simulations, we investigate mass- and luminosity-normalized specific frequencies (T_N and S_N , respectively) of globular cluster systems (GCSs) in order to understand the origin of the observed Ushaped relation between S_N and V-band magnitude (M_V) of their host galaxies. We adopt a biased GC formation scenario in which GC formation is truncated in galaxy halos that are virialized at a later redshift, z_{trun} . T_{N} is derived for galaxies with GCs today and converted into S_N for reasonable galaxy mass-tolight-ratios (M/L). We find that T_N depends on halo mass (M_h) in the sense that $T_{\rm N}$ can be larger in more massive halos with $M_{\rm h} > 10^9 {\rm M}_{\odot}$, if $z_{\rm trun}$ is as high as 15. We however find that the dependence is too weak to explain the observed $S_{\rm N}$ - $M_{\rm V}$ relation and the wide range of $S_{\rm N}$ in low-mass early-type galaxies with $-20.5 < M_{\rm V} < -16.0$ mag for a reasonable constant M/L. The $M_{\rm V}$ -dependence of $S_{\rm N}$ for the low-mass galaxies can be well reproduced, if the mass-to-light-ratio $M_h/L_V \propto M_h^{\alpha}$, where α is as steep as -1. Based on these results, we propose that the origin of the observed U-shaped $S_{\rm N}$ - $M_{\rm V}$ relation of GCSs can be understood in terms of the bimodality in the dependence of M_h/L_V on M_h of their host galaxies. We also suggest that the observed large dispersion in S_N in low-mass galaxies is due partly to the large dispersion in $T_{\rm N}$.

Subject headings: globular clusters: general – galaxies: star clusters – galaxies: evolution – galaxies: stellar content

1. Introduction

Specific frequencies (S_N) of globular clusters (GCs) in galaxies are observed to be different between galaxies with different Hubble morphological types and between those in different environments (e.g., Harris 1991). The S_N of globular cluster systems (GCSs) and their correlations with physical properties of their host galaxies have long been discussed in various different contexts of galaxy formation; biased GC formation in high-density environments (e.g., West 1993), origin of field and cluster elliptical galaxies (e.g., Ashman & Zepf 1992; Forbes et al. 1997), and origin of dwarf elliptical (dE) galaxies (e.g., Miller et al. 1998). Recent numerical and theoretical studies have also discussed the origin of S_N of elliptical galaxies interacting with cluster environments (e.g., Bekki et al. 2003).

Durrell et al. (1996) investigated the trend between S_N and M_V (where M_V is the V-band magnitude of a galaxy) for galaxies with a wide range of M_V . They found that S_N values were higher in the lowest luminosity dwarf ellipticals (dEs), and similar to the S_N values for giant ellipticals (see Fig. 9 in their paper). S_N values appeared to have a minimum at intermediate galaxy luminosities, but there were very few data points available to them. Forbes (2005) recently investigated the S_N values for \sim 100 early-type galaxies in the Virgo cluster, confirming the high S_N values for dwarfs and low S_N values at $M_V \sim$ –19.5. Thus the S_N - M_V relation reveals a 'U'-shaped, or bimodal, distribution. Durrell et al. (1996) speculated that supernova-driven mass loss was responsible for the higher S_N values in dwarfs. Forbes (2005) suggested that this, and the transition from hot to cold gas accretion (Dekel & Birnboim 2005) played a role in determining the number of GCs per unit galaxy starlight. He further noted that this increase in S_N closely followed the dependence of galaxy M/L with mass. However, this observed S_N bimodality has not yet been explored by theoretical/numerical studies of GC formation.

The purpose of this *Letter* is to demonstrate, for the first time, whether and how the observed U-shaped S_N - M_V relation is reproduced in a "biased GC formation scenario" in which GCs can be formed only in galaxy subhalos with high primordial densities and thus virialized earlier (i.e., further GC formation is truncated after a redshift of z_{trun}). This biased GC formation scenario has been discussed in the context of GCS properties (e.g., West 1993; Beasley et al. 2002; Santos 2003; Bekki 2005; Yahagi & Bekki 2005; Moore et al. 2005), but not discussed in the context of the U-shaped S_N distribution. Throughout this paper, we adopt the standard definition of $S_N = N_{\text{GC}} \times 10^{-0.4(M_V + 15)}$ and $T_N = T_0 N_{\text{GC}}/M$, where N_{GC} is the total number of GCs in a GCS and T_0 is a normalization factor such that T_N has reasonable values (an order of 1 - 10).

2. The observed U-shaped relation and its fitting function

Figure 1 shows the observed U-shaped $S_{\rm N}$ - $M_{\rm V}$ relation from Harris (1991) for bright galaxies and from Durrell et al. (1996) for dwarf ones. It is clear from this figure that the galaxy luminosity dependence of $S_{\rm N}$ is different between galaxies below and above a threshold luminosity, $M_{\rm V,th}$. Although the dependence is less clear for galaxies with $M_{\rm V} < M_{\rm V,th}$ in Figure 1, Rhode et al. (2005) found a strong trend of increasing $T_{\rm N}$ with increasing galaxy luminosity for elliptical galaxies with mass $> 10^{11} {\rm M}_{\odot}$.

The observed S_N - M_V and T_N -M relations may provide vital clues to galaxy and GC formation (e.g., Harris 1991; Forbes 2005). Accordingly comparison between observed and simulated relations may well enable us to derive some physical meaning for the relations. It is therefore important to quantify the relations by using an admittedly simple mathematical expression. We here propose the following form for the S_N - M_V relation;

$$S_{\rm N}(x) = A_1 \times 10^{K_1 x} + (S_{\rm N,th} - A_1) \times 10^{K_2 x},\tag{1}$$

where $x=\frac{M_{\rm V}-M_{\rm V,th}}{M_{\rm V,th}}$ and $S_{\rm N,th}$ is $S_{\rm N}$ at $M_{\rm V}=M_{\rm V,th}$. Parameter values of $A_1,~K_1,~K_2,~M_{\rm V,th}$, and $S_{\rm N,th}$ can be determined by fitting to observations. An example of the model fit with $M_{\rm V,th}=-19.5~{\rm mag},~S_{\rm N,th}=1.0,~A_1=0.5,~K_1=-6,~{\rm and}~K_2=4$ is shown in Figure 1.

This functional form in equation (1) can be equivalent to $B_1 \times L_V^{C_1} + B_2 \times L_V^{C_2}$, where L_V is the V-band luminosity and other parameter values (e.g., B_1) can be derived from values of the parameters (e.g., A_1) in equation (1). Therefore, the functional form has some advantages in discussing both theoretically and observationally the origin of S_N (and T_N) in terms of mass and luminosity growth processes of galaxy assembly. The observed γ of 1-2 in $N_{GC} \propto L_V^{\gamma}$ for bright ellipticals (Zepf & Ashman 1993) suggests that C_2 can range from 0 to 1.

3. Numerical simulations of T_N and S_N

3.1. The model

Since methods and techniques of numerical simulations of GC formation at high redshifts are given in detail in our previous paper (Yahagi & Bekki 2005, YB), we only briefly describe them in this paper. We simulate the large scale structure of GCs in a Λ CDM Universe with $\Omega = 0.3$, $\lambda = 0.7$, $H_0 = 70$ km s⁻¹ Mpc⁻¹, and $\sigma_8 = 0.9$ by using the Adaptive Mesh Refinement N-body code developed by Yahagi (2005) and Yahagi & Yoshii (2001). We use 512³ collisionless dark matter (DM) particles in a simulation with the box size of

 $8.26h^{-1}$ Mpc and a particle mass of $5 \times 10^5 M_{\odot}$. We start simulations at z = 41 and follow them until z = 0 in order to investigate the physical properties of old GCs outside and inside virialized dark matter halos at z = 0. We used the COSMICS (Cosmological Initial Conditions and Microwave Anisotropy Codes), which is a package of fortran programs for generating Gaussian random initial conditions for nonlinear structure formation simulations (Bertschinger 1995, 2001).

We assume that (1) old, metal-poor globular cluster (MPC) formation is truncated for $z \leq z_{\text{trun}}$, and (2) initial radial profiles of GCSs in virialized subhalos at $z = z_{\text{trun}}$ are the same as those of the "NFW" profiles (Navarro, Frenk, & White 1996). We follow dynamical evolution of GCs from $z = z_{\text{trun}}$ to 0 and thereby investigate mass of the virialized halos at z = 0 (M_{h}), the total numbers of GC particles within the halos (N_{GC}), and the normalized number of GCs per halo mass ($T_{\text{N}} = T_0 N_{\text{GC}}/M_{\text{h}}$). Given the fact that there are currently no observational constraints on initial GCS density profiles at $z = z_{\text{trun}}$, we consider that adopting the NFW profiles is a reasonable first step for better understanding GCS properties in the CDM model.

We convert the simulated $T_{\rm N}$ - $M_{\rm h}$ relations into the $S_{\rm N}$ - $M_{\rm V}$ ones by using the following formula:

$$S_{\rm N} = S_0 \frac{N_{\rm GC}}{L_{\rm V}} = S_0 \left(\frac{N_{\rm GC}}{M_{\rm h}}\right) \left(\frac{M_{\rm h}}{L_{\rm V}}\right) = S_0 \left(\frac{T_N}{T_0}\right) \left(\frac{M_h}{L_V}\right) \tag{2}$$

where S_0 is a constant and the normalization factor T_0 for each model is determined such that number of GCs per unit mass in a simulation box is the same between models with different z_{trun} . We then compare the simulated relations to the observed data of Forbes (2005) in which the S_N of blue GCs was derived based on new data from the ACS Virgo Cluster Survey (Peng et al. 2005).

We investigate two M/L models, i.e. variable and constant M/L. For the variable M/L model, we assume that $M_{\rm h}/L_{\rm V} \propto M_{\rm h}{}^{\alpha}$ for $M_{\rm h} \leq M_{\rm h,th}$, where $M_{\rm h,th}$ is set to be $10^{11} {\rm M}_{\odot}$, and $M_{\rm h}/L_{\rm V} \propto M_{\rm h}{}^{\beta}$ for $M_{\rm h} > M_{\rm h,th}$. We here focus mainly on low-luminosity galaxies for which observational data of $S_{\rm N}$ is available (Forbes 2005). We accordingly chose the value of β consistent with observations based on galaxy luminosity functions (i.e. $\beta=0.33$; Marinoni & Hudson 2002) and derive the best value of α for low-luminosity galaxies. Although we investigate models with a wide range of $M_{\rm h}/L_{\rm V}$ (i.e. 5-100) for the constant M/L model, we only show the results of the model with $M_{\rm h}/L_{\rm V}=10$. It should be stressed here that all of the present results are galaxies containing GCs at z=0; GC-less galaxies at z=0 (typically those with $M_{\rm V}>-15$) are not included in the present study.

Although Forbes (2005) speculated about a possible M-dependent M/L explaining the observed S_N - M_V relation, his adopted assumptions (e.g., constant T_N) are not based on

detailed theoretical/numerical studies. Since the present study derives $S_{\rm N}$ from the simulated $T_{\rm N}$, we can discuss more self-consistently the origin of the $S_{\rm N}$ - $M_{\rm V}$ relation on firm physical basis. We demonstrate that the simulated $T_{\rm N}$ - $M_{\rm h}$ dependences are too weak to explain the wide range of $S_{\rm N}$ for a constant M/L: The difference in GC number per unit galaxy mass between galaxies alone can not explain observations.

3.2. Results

Figure 2 shows the distributions of the simulated GCSs at z=0 in the $T_{\rm N}$ - $M_{\rm h}$ plane for the models with $z_{\rm trun}=6$ and 15. Below a critical halo mass of $\sim 2\times 10^9{\rm M}_\odot$, $T_{\rm N}$ is higher for halos with smaller $M_{\rm h}$ in the model with $z_{\rm trun}=15$. Above the critical mass, $T_{\rm N}$ is higher for halos with larger $M_{\rm h}$ in this model and the slope of the $M_{\rm h}$ -dependence of $T_{\rm N}$ is significantly shallower than in halos below $M_{\rm h,th}$. This clear bimodality in the $T_{\rm N}$ - $M_{\rm h}$ relation is not seen in the model with $z_{\rm trun}=6$, which shows a monotonically increasing trend of $T_{\rm N}$ with $M_{\rm h}$ for $10^8{\rm M}_\odot \leq M_{\rm h} \leq 3\times 10^{10}{\rm M}_\odot$. The dispersion in $T_{\rm N}$ appears to be significantly larger in halos with smaller $M_{\rm h}$ for both models, which implies that the observed large dispersion in $S_{\rm N}$ in low-mass galaxies can be understood in terms of the large dispersion in $T_{\rm N}$.

For the model with $z_{\rm trun}=15$, GC formation is assumed to occur only in the subgalactic halos formed from rare high-density peaks in the primordial matter distribution and thus virialized before z=15. Accordingly, the bimodality can be understood in terms of the following two competing effects of halo merging with, or without, GCs. Low-mass subhalos with initially high $T_{\rm N}$ (virialized at z>15) grow after their formation by merging/accretion of subhalos most of which are virialized later than z=15 and thus are those without GCs (referred to as "GC-less" halos). Consequently, their $T_{\rm N}$ values can decrease as their masses become larger during hierarchical growth. This is the first effect of halo merging.

According to the theory of biased galaxy formation (e.g., Kaiser 1984), the primordial density field of a more massive halo today is more likely to contain a larger fraction of high-density peaks. Accordingly, a larger halo at z=0 can be formed from a larger fraction of subhalos that are virialized before z=15 and which contain GCs ("GC" halos). Therefore, a larger halo is more likely to be formed from merging of a larger fraction of GC halos and thus merging can increase $T_{\rm N}$ for larger halos. This is the second effect of halo merging.

As a result of these two competing effects, we can understand the bimodal $T_{\rm N}$ - $M_{\rm h}$ relation, i.e. below $M_{\rm h,th}$ the first effect is stronger, while above the second effect is stronger. The dramatic effect of the first GC-less halo merging is not expected for the model with $z_{\rm trun}=6$, because most of subgalactic halos that are building blocks of larger halos at z=0

have been virialized before z=6 and contain GCs. Therefore the monotonically increasing $T_{\rm N}$ - $M_{\rm h}$ relation is due to the second effect in this model. It should be stressed here that irrespective of $z_{\rm trun}$, $T_{\rm N}$ is more likely to be larger for halos with $M_{\rm h} > 10^9 {\rm M}_{\odot}$.

Figure 3 shows the comparison between the observations (Forbes 2005) and the best variable M/L model with $\alpha \approx -1$, where α is the slope in the relation of $M_{\rm h}/L_{\rm V} \propto M_{\rm h}{}^{\alpha}$. It should be noted here that the observed $S_{\rm N}$ of blue GCs in ellipticals brighter than $M_{\rm V} \sim -20.5$ mag are seriously underestimated due to the limited ACS field-of-view (Forbes 2005). Thus the comparison makes sense only for lower luminosity galaxies. As shown in Figure 3, both models with $z_{\rm trun} = 6$ and $z_{\rm trun} = 15$ can reproduce well the observed trend of increasing $S_{\rm N}$ with decreasing luminosity for low-mass galaxies with $M_{\rm V} > M_{\rm h,th}$.

Figure 4 shows that the simulated $S_{\rm N}$ - $M_{\rm V}$ relations do not match well with the observations for the constant M/L models. We also find that constant $M_{\rm h}/L_{\rm V}$ models, with a wide range of $M_{\rm h}/L_{\rm V}$ values (i.e. 5 – 100), do not match with the observations either. These results, combined with those in Figure 3, imply that the origin of the $S_{\rm N}$ - $M_{\rm h}$ relation is closely associated with the dependence of $M_{\rm h}/L_{\rm V}$ on $M_{\rm h}$, as suggested by Forbes (2005). Figure 4 shows that $S_{\rm N}$ can be only slightly higher in low-luminosity galaxies with $M_{\rm V} > -16.5$ mag as a result of biased GC formation in the model with $z_{\rm trun} = 15$. However the simulated dependence is not strong enough to explain quantitatively the observation for the constant $M_{\rm h}/L_{\rm V}$ model. Since observations for galaxies brighter than $M_{\rm V} = -16$ mag should be compared with simulations, the results for $M_{\rm h} > 2 \times 10^9 {\rm M}_{\odot}$ in Figure 2 are relevant to Figure 4. The derived $S_{\rm N}$ - $M_{\rm V}$ relations in Figure 4 are thus essentially the same as $T_{\rm N}$ - $M_{\rm h}$ ones for $M_{\rm h} > 2 \times 10^9 {\rm M}_{\odot}$ in Figure 2. If we adopt a higher M/L, the blue/red lines are shifted to the left in Figure 4: A model with M/L = 25 results in a 1 mag shift to lower luminosities.

4. Discussion and conclusions

West (1993) first suggested that the observed dependence of $S_{\rm N}$ on galaxy type and environment can be understood in terms of GC formation from the statistical enhancement of rare high-density peaks in the primordial matter distribution (i.e., biased GC formation). The present numerical simulations have shown that biased GC formation in subgalactic halos virialized early ($z \sim 15$), can be imprinted on the observed $T_{\rm N} - M_{\rm h}$ relation. Semi-analytic galaxy formation models based on a hierarchical clustering scenario have also shown that models with a truncation of GC formation at $z \sim 5$ can better reproduce the observed color bimodality of GCs in early-type galaxies (Beasley et al. 2002). What physical mechanisms are responsible for the truncation (or the severe suppression) of GC formation in subgalactic halos virialized at later times?

Santos (2003) first proposed that cosmic reionization at high z can suppress GC formation and thus be an important factor for better understanding the origin of the physical properties of GCSs. Recent numerical simulations on GCs and old stellar halos in galaxies have confirmed that structural and kinematical properties of GCSs carry fossil information about the epoch of the reionization z_{reion} that can suppress GC formation (Bekki 2005; Bekki & Chiba 2005; Moore et al. 2005; Rhode et al. 2005). Therefore, if $z_{\text{trun}} = z_{\text{reion}}$, the present results imply that the $T_{\text{N}} - M_{\text{h}}$ relation can provide some constraints on z_{reion} . Furthermore, we suggest that these relations can be different in different environments (or cosmic volumes investigated by observations), if reionization is spatially inhomogeneous (Benson et al. 2001; Wyithe & Loeb 2004).

We suggest that it is worthwhile for future observations to investigate the $T_{\rm N}$ - $M_{\rm h}$ relation for a statistically significant number of galaxies, although it is a formidable task to derive total galactic masses including dark matter halos. Such observational studies can test biased GC formation at high redshifts by comparing the results with the corresponding simulations. The predicted $T_{\rm N}$ values change only by a factor of ~ 5 for a change in $M_{\rm h}$ by four orders of magnitude. The observed difference in $S_{\rm N}$ for a $M_{\rm V}$ range of ~ 10 mag (corresonding roughly to a luminosity difference of factor $\sim 10^4$) is more than a factor of ~ 30 (see Figure 1). Therefore the observed bimodal $S_{\rm N}$ - $M_{\rm V}$ relation can not be explained well by the present models without resorting a mass-dependent M/L (for which there is some independent evidence). We accordingly suggest that the origin of the observed U-shaped $S_{\rm N}$ - $M_{\rm V}$ relation of GCSs can be associated, at least partly, with the bimodality in the mass-dependence of the M/L ratio of their host galaxies.

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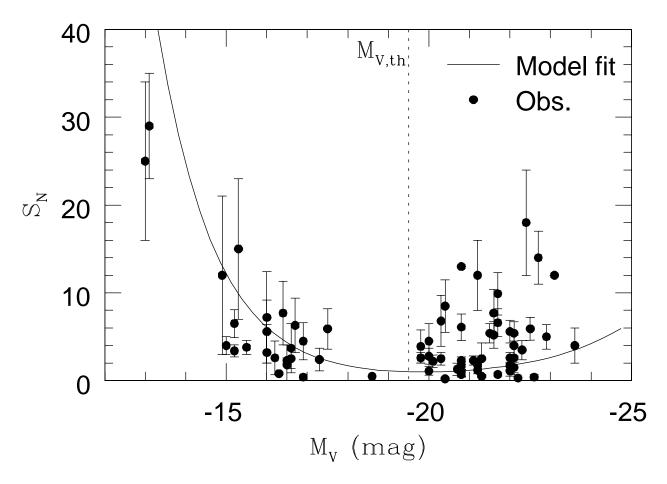


Fig. 1.— Observational data for GCSs in galaxies in the $S_{\rm N}$ - $M_{\rm V}$ plane. Observational data are from Harris (1991) for bright galaxies and from Durrell et al. (1996) for dwarf populations. The solid line represents an empirical fit to the observed bimodality in the relation between $S_{\rm N}$ and $M_{\rm V}$ (the details of the functional form are given in the main text). The dotted line marks the critical magnitude ($M_{\rm V,th}$): The $S_{\rm N}$ dependence on $M_{\rm V}$ is different between low and high luminosity galaxies.

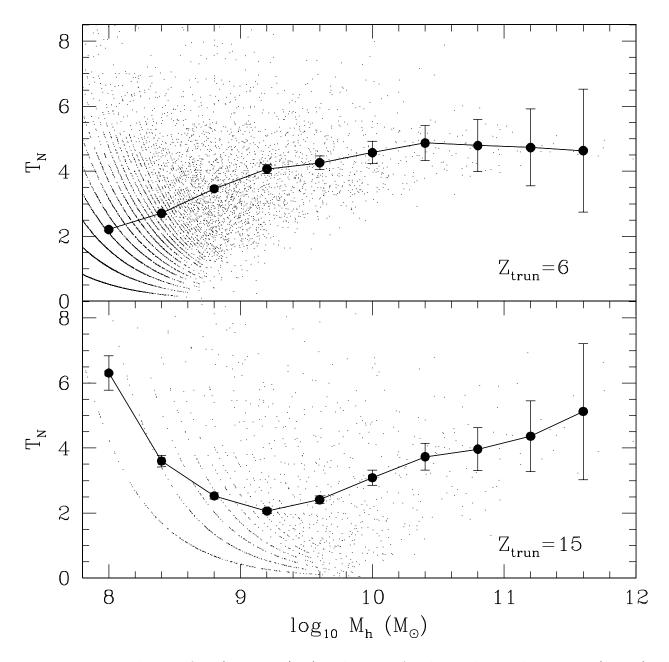


Fig. 2.— Dependences of $T_{\rm N}$ (= $T_0N_{\rm GC}/M_{\rm h}$) on $\log_{10}M_{\rm h}$ for the models with $z_{\rm trun}=6$ (upper) and $z_{\rm trun}=15$ (lower). Small dots represent simulated GCSs within galaxy-scale halos of masses $M_{\rm h}$ and "GC particle" numbers of $N_{\rm GC}$. Big dots with error bars represent the mean values of $T_{\rm N}$ in each mass bin. The error bar in each mass bin is estimated as $T_{\rm N}/\sqrt{2(N_i-1)}$, where N_i is the total number of galaxy-scale halos in the i-th mass bin. In order to compare these results with observations, the value of the normalization factor (T_0) is determined for each model. See text for details.

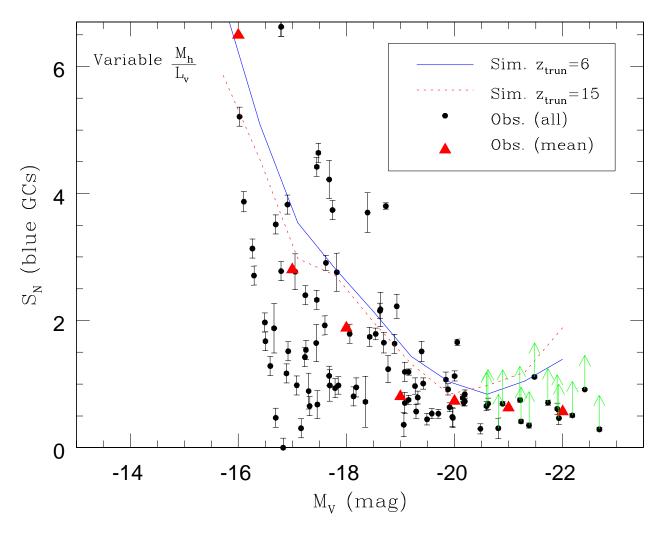


Fig. 3.— Observational data for ~ 100 GCSs in early-type galaxies in the $S_{\rm N}$ - $M_{\rm V}$ plane. Here $S_{\rm N}$ values are shown for blue GCs only from Forbes (2005) and the large triangles show the mean value of $S_{\rm N}$ in each magnitude bin. For comparison, the variable M/L models with $M_{\rm h}/L_{\rm V} \propto M_{\rm h}^{-1.0}$ are shown for $z_{\rm trun}=6$ (solid) and $z_{\rm trun}=15$ (dotted). Since $S_{\rm N}$ of brighter galaxies with $M_{\rm V}<-20.5$ mag are significantly underestimated in observations, the lower limits of $S_{\rm N}$ values are indicated by green arrows for these galaxies.

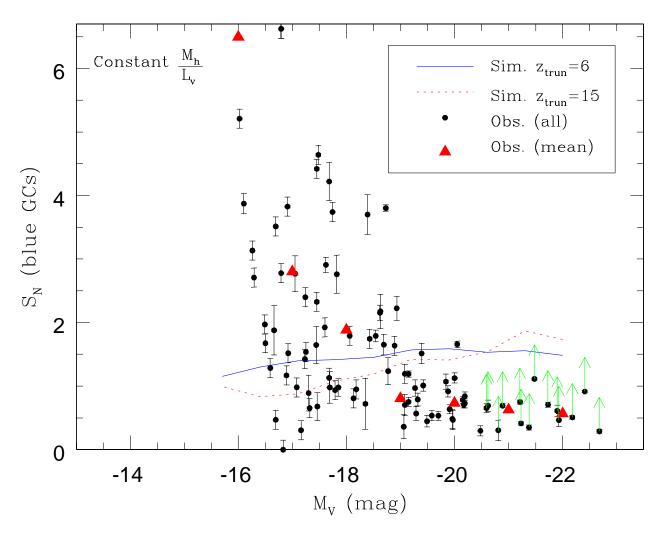


Fig. 4.— The same as Figure 3 but for the models with constant $M_{\rm h}/L_{\rm V}$ (=10). The shapes of the derived $S_{\rm N}-M_{\rm V}$ relation can be slightly different from those of the $T_{\rm N}-M_{\rm h}$ one shown in Figure 2, though overall shapes are quite similar between the two relations for $M_{\rm h}>8.5\times10^8{\rm M}_{\odot}$ (corresponding to $M_{\rm V}<-15.0$ mag). The origin of the slight differences is associated with the difference in the binning process of simulation data between Figures 2 and 4.